PERFORMANCE OF UNSTABLE PENDANT DROPS IN SURFACE TENSION MEASUREMENTS

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ABSTRACT· Tawde and Parvatikar have evolved an epxerimental technique to subject the unstable pendant drops for surface tension measurements. In this paper modifying this technique and using a table drawn from fundamental considerations utilising Fordham's tables, surface tension measurements have been made from the observed data on four liquids which compare favourably with the accepted values.

INTRODUCTION

The use of pendant drops for surface tension measurements has been suggested by several workers such as Worthington (1885) and Ferguson (1912). The method remained in disrepute for a considerable time because of difficult measurements involved in it. Recently, it has been made useful for exact work as a result of critical study of it by Andreas, *et al.* (1938). This method has been placed on better foundations by Fordham (1948) by supplying a table for the calculation of surface tension from measurements on pendant drops. Brown and McCormick (1948), while working out a new drop-weight method have shown by dimensional analysis that the shapes of all drops forming on a conical tip are similar at the unstable stage.

As shown by Adam (1941), the equation to the outline of any pendant drop referred to axes of x and z can be written in the dimensionless form

$$\frac{1}{\rho/b}+\frac{\sin\phi}{x/b}=2+\beta\,\frac{z}{b} \qquad \ldots \quad (1)$$

where b is the radius of curvature at the origin, which is the vertex of the drop, ρ is the radius of curvature in the plane of the paper at the point (x, z), and $\beta=\frac{2b^2}{a^2}$, where a^2 is the capillary constant which connects the surface tension γ by the relation $a^2=\frac{2\gamma}{g\sigma}$, σ being the effective density of the liquid. The angle ϕ is the inclination at the point (x, z) to the horizontal.

Bashforth and Adams (1883), giving numerical solution of this equation have drawn up a table of x/b and z/b for many values of ϕ and β. These tables were extended by Fordham while supplying a tabular set of values necessary in the pendant drop method suggested by Andreas, Hauser and Tucker.

It is shown possible here that for a given value of β, the ratio d/r, $d(=z_e)$, being the depth of a pendant drop from the equatiorial plane and $r(=x_e)$ the radius of it in that plane, may be related to a^2/r^2. Obviously $\phi = 90°$, when the point lies on the equatorial plane.

Now,
$$\frac{d}{r} = \frac{z_e}{x_e} = \frac{z_e/b}{x_e/b} \qquad \ldots \quad (2)$$

and
$$a^2 = \frac{2b^2}{\beta} \quad \left(\text{since } \beta = \frac{2b^2}{a^2}\right)$$

therefore,
$$\frac{a^2}{r^2} = \frac{a^2}{x_e^2} = \frac{2}{\beta(x_e/b)^2}\,. \qquad \ldots \quad (3)$$

Knowing the values of z_e/b and x_e/b by interpolation from the tables of Bashforth and Adams, and Fordham, it is possible to calculate d/r from Eq. (2) and a^2/r^2 from Eq.(3). The computed values necessary for the present problem are given in Table I. This table allows for direct interpolation of the intermediate values of d/r.

TABLE I

β	Z_e/b	X_e/b	d/r	a^2/r^2	Interpolated values	
					d/r	a^2/r^2
0.4500	1.309,01	1.103,67	1.186,05	3.648,71	1.186	3.648,71
0 4625	1.327,12	1.108,00	1.197,76	3 522,40	1.197	3.529,95

If it is possible to measure d/r of a pendant drop formed on a conical tip at the stage of instability, a^2/r^2 can be known for the corresponding measured value of d/r, and hence γ, the surface tension can be calculated. It may be noted here, as mentioned above, that since the shapes of all drops forming on a conical tip are similar at the unstable stage, only one value of a^2/r^2 is required to be known precisely for an accurately measured value of d/r.

The experimental problem was, therefore, of measuring the equatorial radius r, and the depth d of a pendant drop formed on a conical tip at the critical stage of instability. If sufficient time is allowed for proper development of a drop formed on a conical tip, it is possible to follow the changing shape and size of the drop until it just collapses from it,

EXPERIMENTAL AND RESULTS

The experimental set up devised and the operations involved were exactly the same as adopted by Tawde and the author (1956), with the following modifications. The source S was a monochromatic radiation of sodium lamp and the microscope was replaced by a camera, its lens being fitted in a tube which was fixed to the front window of the thermostat chamber. The camera was set on an optical bench at a suitable distance in order to get an enlarged sharp image of the drop. There was a channel connecting the tube in which lens was inserted and the camera. In the place of the photographic plate in the camera, an oiled paper was enclosed in between two thin plane glass plates A weighted silk thread was hung as a plumb line very near to the glass plate and its image defined the vertical. A sharp blade was fixed to a fine screw arrangement which was fitted to the frame of the camera. By turning the screw, it was possible to move the edge of the blade up and down even to a very small extent. The image of the edge of the blade was perfectly horizontal.

As the method involved measurements on the images of drops and the actual radius of the drop in the equatorial plane, it was necessary to know the magnification ratio (M). This was achieved by measuring the base of the actual conical tip and its enlarged image. The base of the conical tip was 12.60 ± 0.02 mm. By loosening the pinch-cock screw, a drop was allowed to grow slowly under gravity at the conical tip. The time taken for a drop to develop fully to the point of detachment was atleast seven minutes. The image of the plumb line was first adjusted so that the edge of the conical tip coincided with it. At the unstable stage of the pendant drop, the point lying on the equatorial plane was observed by a microscope capable of reading 0.001 cm. and at the same time by turning the screw the image of the edge of the blade was set tangential to the bottom of the drop. From these two settings on the image of the drop, $D/R(=d/r)$ was obtained. Readings were repeated a number of times on fresh drops to obtain

TABLE II

Liquid at 30°C	Effective density in gm./cc.	M	D in mm.	R in mm.	$\frac{D}{R}$	r in mm.	$\frac{a^2}{r^2}$ (From Table I)	a^2 in sq. mm.	γ in dynes/cm. Present author	I.C.T.
Water	0.9946	5.054	12.29	10.27	1.196	2.032	3.540,75	14.620	71.14	71.18 $\pm$0.05
Toluene	0.8525	7.684	12.49	10.44	1.196	1.358	3.540,75	6.529	27.23	27.30 $\pm$0.10
Benzene	0.8622	7.685	12.50	10.45	1.196	1.360	3.540,75	6.548	27.62	27.60 $\pm$0.05
m-xylene	0.8530	7.698	12.62	10.55	1.196	1.370	3.540,75	6.646	27.74	27.80 $\pm$0.10

confirmation of the ratio D/R. The actual radius r of the drop was computed knowing R and M. The liquids chosen for surface tension measurements were water, toluene, benzene and *m*-xylene. A sample set of observations which are the mean of at least ten independent drops of each liquid which did not vary appreciably from each other are given in Table II. No attempt has been made at this stage to obtain the estimate of accuracy of results.

On comparing the results in the last two columns of Table II, it is apparent that there is a fair agreement of measured values of surface tension with those of I.C.T. Furthermore, since the shape factor d/r is the same for drops of different liquids formed on a conical tip, the investigation shows that the shapes of *all* drops forming on a conical tip are similar at the unstable stage—a conclusion reached by Brown and McCormick by dimensional analysis. It would be interesting to examine how far the unstable pendant drops can be used to measure the surface tension of liquids on this basis. This point is under investigation in this laboratory.

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REFERENCES

Adam, N. K., 1941, *The Physics and Chemistry of surfaces*, Oxford University Press, p. 366.

Andreas, J. M., Hauser, E. A and Tucker, W. B., 1938, *J Phy. Chem*, **42**, 1001.

Bashforth, F. and Adams, J. C., 1883, *An attempt to test the theories of capillary action.* Cambridge University Press

Brown, R. C. and McCormick, H., 1948, *Phil. Mag.*, **39**, 420.

Ferguson, A., 1912, *Phil. Mag.*, **23**, 417.

Fordham, S., 1948, *Proc. Roy Soc.*, **194**, 1.

Tawde, N R. and Parvatikar, K. G., 1956, *Ind J Phys*, **30**, 348.

Worthington, A. M., 1885, *Phil. Mag.*, **19**, 46.